

**CAMBRIDGE INTERNATIONAL EXAMINATIONS**

**GCE Advanced Level**

## **MARK SCHEME for the May/June 2014 series**

### **9231 FURTHER MATHEMATICS**

**9231/23**

Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2014 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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### Mark Scheme Notes

Marks are of the following three types:

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep\*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol  $\nabla$  implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.  
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking  $g$  equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a "fortuitous" answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

### **Penalties**

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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Question Number	Mark Scheme Details	Part Mark	Total
1 (i)	<p>Use conservation of momentum, e.g. <math>mv_A + kmv_B = mu + \frac{2}{3}kmu</math></p> <p><i>or</i> <math>v_A + kv_B = u(1 + \frac{2}{3}k)</math> B1</p> <p>Use restitution (4/5 on wrong side is M0; signs inconsistent with prev. eqn is A0): <math>v_A - v_B = -(4/5)(u - \frac{2}{3}u)</math></p> <p><i>or</i> <math>v_A - v_B = -4u/15</math> M1 A1</p> <p>Solve for <math>v_A</math> (allow verification): <math>(1 + k)v_A = u(1 + \frac{2}{3}k - 4k/15)</math></p> <p><math>v_A = u(2k + 5)/5(k + 1)</math> <b>A.G.</b> M1 A1</p> <p><math>[v_B = u(10k + 19)/15(k + 1)]</math></p>	5	
(ii)	<p>Equate impulse to momentum change for A: <math>mu - (2/5)mu = mv_A</math></p> <p><math>3/5 = (2k + 5)/5(k + 1), k = 2</math> M1 A1</p> <p><i>OR B:</i> <math>\frac{2}{3}kmu + (2/5)mu = kmv_B</math></p> <p><math>\frac{2}{3}k + (2/5) = k(10k + 19)/15(k + 1)</math></p> <p><math>10k^2 + 16k + 6 = 10k^2 + 19k, k = 2</math> (M1 A1)</p>	2	7

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2	Find radial acceleration from $r (d\theta/dt)^2$ [ $\equiv r\omega^2$ ]: $d\theta/dt = (2 \sin 2t) (2 \cos 2t)$ $= 2 \sin 4t$ $r (d\theta/dt)^2 = 4a \sin^2 4t$ <b>A.G.</b> M1 A1	2	
	(i) Find $t$ by equating $d^2\theta/dt^2$ to 0: $d^2\theta/dt^2 = 8 \cos 4t = 0$ $t = \pi/8$ or 0.393 M1 A1	2	
	(ii) Find radial and transverse components of force: $4ma \sin^2 4\pi/12 = 3ma$ and $8ma \cos 4\pi/12 = 4ma$ Combine to find magnitude of resultant force: $\sqrt{(3^2 + 4^2)} ma = 5ma$ M1 A1	2	<b>6</b>

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3	(i)	Use conservation of energy:	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mg(a - a \cos \theta)$ B1	5	
		Use $F = ma$ radially:	$T - mg \cos \theta = mv^2/a$ B1		
		Relate $u$ to impulse $J$ :	$mu = J$ B1		
		Eliminate $u$ and $v$ to find $T$ :	$T = J^2/ma - mg(2 - 3 \cos \theta)$ A.G. M1 A1		
	(ii)(a)	Investigate $v$ [and $T$ ] for $k = 1$ and describe motion:	$v^2 = ga(2 \cos \theta - 1)$ $[T = mg(3 \cos \theta - 1)]$ $v = 0$ [and $T > 0$ ] when $\cos \theta = \frac{1}{2}$	2	
		(S.R. Award B1 for correct result based only on $T$ )	so $P$ oscillates (A.E.F.) M1 A1		
	(b)	Investigate $v$ and $T$ for $k = 6$ and describe motion:	$T = mg(3 \cos \theta + 4)$ $[v^2 = ga(2 \cos \theta + 4)]$ $T > 0$ for e.g. $\theta = \pi$ and $v > 0$	2	9
		(S.R. Award B1 for correct result based only on $T$ )	so $P$ does full circle(A.E.F.) M1 A1		

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4	<i>EITHER:</i> Resolve vertically:	$R_A + R_C \cos \alpha = 3mg/2$	B1		
	Take moments about A:	$R_C 2a = mgd \cos \alpha + \frac{1}{2}mg2d \cos \alpha$	M1 A1		
	Eliminate $R_C$ to find $R_A$ :	$R_C = (mgd/a) \cos \alpha = 3mgd/5a$			
		$R_A = 3mg/2 - 9mgd/25a$ or $3mg(25a - 6d)/50a$ (A.E.F.)	M1 A1		
	<i>OR:</i> Resolve along AB:	$R_A \sin \alpha + F_A \cos \alpha =$ $mg \sin \alpha + \frac{1}{2}mg \sin \alpha$ (B1)			
	Take moments about C:	$(R_A \cos \alpha) 2a - (F_A \sin \alpha) 2a =$ $(mg \cos \alpha)(2a - d)$ $- (\frac{1}{2}mg \cos \alpha)(2d - 2a)$	(M1 A1)		
	Eliminate $F_A$ to find $R_A$ :	$R_A = 3mg(25a - 6d)/50a$ (A.E.F.)	(M1 A1)		
Find limit on $d$ from $R_A > 0$ :	$25a - 6d > 0, d < 25a/6$	A.G. B1	6		
Find $F_A$ by e.g. horizontal resolution:	$F_A = R_C \sin \alpha = (3mgd/5a)(4/5)$ $= 12mgd/25a$	M1 A1			
Find inequality for $\mu$ from $F_A \leq \mu R_A$ (= loses A1):	$\mu \geq 8d/(25a - 6d)$	A.G. M1 A1	4	10	

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5	<p>Find MI of rectangular lamina about <math>O</math>: <math>I_{\square,O} = \frac{1}{3}M \{(4a)^2 + (3a)^2\}</math> B1  <math>[= 25 Ma^2/3]</math></p> <p>Find MI of circular lamina about <math>O</math>: <math>I_{O,O} = \frac{1}{2} \frac{1}{3}M(5a/2)^2</math> B1  <math>[= 25 Ma^2/24]</math></p> <p><i>EITHER:</i> Find MI of combined laminae about <math>O</math>: <math>I_O = I_{\square,A} + I_{O,A} = 225 Ma^2/24</math> M1 A1</p> <p>Find MI of combined laminae about <math>A</math>: <math>I_A = I_O + (4M/3) 25a^2</math>  <math>= 1025 Ma^2/24</math> M1 A1</p> <p><i>OR:</i> Find MI of rectangle about <math>A</math> <math>I_{\square,A} = I_{\square,O} + 25 Ma^2 = 100 Ma^2/3</math>  and of circle about <math>A</math>: <i>and</i> <math>I_{O,A} = I_{O,O} + \frac{1}{3}M 25a^2 = 75 Ma^2/8</math> (M1 A1)</p> <p>Find MI of combined laminae about <math>A</math>: <math>I_A = I_{\square,A} + I_{O,A} = 1025 Ma^2/24</math> (M1 A1)</p> <p>Find MI of system about <math>A</math>: <b>A.G.</b> <math>I = I_A + 50 Ma^2 = 2225 Ma^2/24</math> M1 A1</p> <p>State or imply that speed is max when <math>AC</math> vertical M1</p> <p>Use energy when <math>AC</math> vertical (or at general point):  <math>\frac{1}{2} I\omega^2 = 4Mg/3 \times 5a + \frac{1}{2}Mg \times 10a</math>  <i>or</i> <math>11Mg/6 \times 70a/11 [=35Mga/3]</math> M1 A1</p> <p>Substitute for <math>I</math> and find <math>k</math> in max ang. speed <math>\omega</math>:  <math>\omega^2 = (112/445)g/a, k = 0.502</math> A1</p>	8	4	12
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7	(i)	Find $P(N = 5)$ :	$P(N = 5) = \left(\frac{3}{4}\right)^4 \times \frac{1}{4}$ $= 81/1024 \text{ or } 0.079[1]$	B1	1	
	(ii)	Find $P(N > 8)$ :	$P(N > 8) = \left(\frac{3}{4}\right)^8$ $= 6561/65536 \text{ or } 0.1[00]$	M1 A1	2	
		Find prob. $P_J$ that James qualifies:	$P_J = P(N \leq 6) = 1 - \left(\frac{3}{4}\right)^6$ $\text{or } \left\{1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \left(\frac{3}{4}\right)^4 + \left(\frac{3}{4}\right)^5\right\} \frac{1}{4}$ $= 3367/4096 \text{ or } 0.822$	M1 A1	2	
		Find prob. $P_C$ that Colin qualifies:	$P_C = 1 - \left(\frac{2}{3}\right)^6 [= 0.9122]$	B1		
		Find prob. that exactly one qualifies:	$P_J(1 - P_C) + P_C(1 - P_J)$ $= (3367/4096)(64/729)$ $+ (665/729)(729/4096)$ $= 0.0722 + 0.1624$ $= 0.235 \text{ (allow } 0.234) \text{ M1 A1}$		3	<b>8</b>

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<b>8</b>	<p>Find expected values (to 1 d.p.):</p> <p style="text-align: right;">32.4    15.3    12.3</p> <p style="text-align: right;">40.5    19.125    15.375</p> <p style="text-align: right;">35.1    16.575    13.325    M1 A1</p> <p>State (at least) null hypothesis (A.E.F.):    <math>H_0</math>: Car type is independent    B1</p> <p>Calculate <math>\chi^2</math> (to 1 d.p.):</p> <p style="text-align: right;"><math>\chi^2 = 0.005 + 1.208 + 1.796</math></p> <p style="text-align: right;">+ 0.500 + 1.243 + 5.716</p> <p style="text-align: right;">+ 0.479 + 0.020 + 1.640</p> <p style="text-align: right;">= 12.6 (allow 12.7)    M1 A1</p> <p>State or use correct tabular <math>\chi^2</math> value (to 3 s.f.):    <math>\chi_{4, 0.95}^2 = 9.488</math> OR 9.49    B1</p> <p>Valid method for reaching conclusion:    Reject <math>H_0</math> if <math>\chi^2 &gt;</math> tabular value    M1</p> <p>Conclusion consistent with correct values (A.E.F.):    Car type is dependent on age    A1</p>	8	<b>8</b>
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9	Find $k$ for which $P(X \geq k) = 0.6$ :	$0.6 = 1 - F(k)$	M1	3	
		$= 1 - (k/8 - 1/4)$	M1		
		$k = 26/5$ or $5.2$	A1		
	Find $G(y)$ from $Y = 2 \ln X$ for $2 \leq x \leq 10$ :	$G(y) = P(Y < y) = P(2 \ln X < y)$		3	
	(allow $<$ or $\leq$ throughout)	$= P(X < e^{y/2}) = F(e^{y/2})$			
	(result may be stated)	$= e^{y/2}/8 - 1/4$ ( $2 \ln 2 \leq y \leq 2 \ln 10$ )			
	or $(\ln 4 \leq y \leq \ln 100)$				
	or $(1.39 \leq y \leq 4.61)$	M1 A1			
State $G(y)$ for other values of $x$ :	$0$ ( $y < 2 \ln 2$ ) and $1$ ( $y > 2 \ln 10$ )		B1	3	
Find $g(y)$ for $2 \ln 2 \leq y \leq 2 \ln 10$ :	$g(y) = e^{y/2}/16$		M1 A1		
Sketch positive exponential for $2 \ln 2 \leq y \leq 2 \ln 10$			B1		
Show $g(y) = 0$ on either side of this interval			B1	4	<b>10</b>

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10	(i)	Calculate sample mean:	$\bar{x} = 132/8 = 16.5$	M1		
		Estimate population variance:	$s^2 = (2192.06 - 132^2/8)/7$ (allow biased here: $1.757$ or $1.326^2$ )	$= 703/350$ or $2.009$ or $1.417^2$		
		State hypotheses (A.E.F.; B0 for $\bar{x}$ ):	$H_0: \mu = 15.8, H_1: \mu > 15.8$	B1		
		Calculate value of $t$ (to 3 s.f.):	$t = (\bar{x} - 15.8)/(s/\sqrt{8}) = 1.4[0]$	M1 A1		
		State or use correct tabular $t$ -value (to 3 s.f.):	$t_{7,0.9} = 1.41[5]$	B1		
		(or can compare $\bar{x}$ with $15.8 + 0.709 = 16.51$ )				
		Consistent conclusion (A.E.F, ✓ on both $t$ -values):	[Accept $H_0$ :] Popn. mean not greater than 15.8	B1 ✓	7	
	(ii)	Find confidence interval (allow $z$ in place of $t$ ) e.g.:	$16.5 \pm t\sqrt{(2.009/8)}$	M1 A1		
		(Use of 15.8 does not lose M1)				
		Use of correct tabular value:	$t_{7,0.975} = 2.36[5]$	A1		
		Evaluate C.I. correct to 3 s.f.:	$16.5 \pm 1.18[5]$ or $[15.3, 17.7$	A1	4	11

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11 A	(i)	<p><i>EITHER:</i> Find extension <math>e</math> at equilibrium point: <math>e = 5l/7</math> B1</p> <p>Resolve forces at eq. pt. to find modulus <math>\lambda</math>: <math>\lambda e/l = mg, \lambda = 7mg/5</math> A.G. B1</p> <p><i>OR:</i> Use conservation of energy to find <math>\lambda</math>: <math>\frac{1}{2}\lambda\{(6l/7)^2 - (4l/7)^2\}/l = mg2l/7</math></p> <p><math>\lambda = 7mg/5</math> A.G. (M1 A1)</p>	2
	(ii)	<p>Use Newton's Law at general point: <math>m d^2x/dt^2 = mg - \lambda(e+x)/l</math></p> <p>[ or <math>-mg + \lambda(e-x)/l</math> ] M1 A1</p> <p>Simplify to give standard SHM eqn: <math>d^2x/dt^2 = -\lambda x/lm</math> or <math>-7gx/5l</math> A1</p> <p>S.R.: Stating this without derivation (max 2/4): (B1)</p> <p>Find period <math>T = 2\pi/\omega</math> with <math>\omega^2 = 7g/5l</math>: <math>T = 2\pi/\sqrt{7g/5l}</math> or <math>2\pi\sqrt{5l/7g}</math> B1</p>	4

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(iii)	<p><i>EITHER:</i> Equate <math>P</math>'s speed to one-half its max. speed:</p> $\omega^2(l^2/7^2 - x^2) = \frac{1}{4} \omega^2 l^2/7^2,$ <p style="text-align: right;">M1 A1</p> <p>Find <math>x</math>: (A.E.F.) <math>x^2 = \frac{3}{4} l^2/7^2, x = (\sqrt{3}/14)l</math> A1</p> <p><i>either:</i> Find <math>t</math> from <math>x = x_0 \cos \omega t</math>: <math>t = \omega^{-1} \cos^{-1} \{(\sqrt{3}/14)l/(l/7)\}</math> M1 A1</p> $= \omega^{-1} \cos^{-1}(\sqrt{3}/2) = \pi/6\omega$ $= (\pi/6) \sqrt{(5l/7g)} \quad \text{A1}$ <p><i>or:</i> Find <math>t</math> from <math>x = x_0 \sin \omega t</math>: <math>t = \frac{1}{4}T - \omega^{-1} \sin^{-1} \{(\sqrt{3}/14)l/(l/7)\}</math> (M1 A1)</p> $= \frac{1}{4}T - \omega^{-1} \sin^{-1}(\sqrt{3}/2)$ $= (\pi/2 - \pi/3) \sqrt{(5l/7g)}$ $= (\pi/6) \sqrt{(5l/7g)} \quad \text{(A1)}$ <p><i>OR:</i> Equate <math>P</math>'s speed to one-half its max. speed:</p> $\omega x_0 \sin \omega t \text{ or } \omega x_0 \cos \omega t = \frac{1}{2} \omega x_0$ <p style="text-align: center;">(M1 A1; A1)</p> <p>Find first value of <math>t</math>:</p> $t = \omega^{-1} \sin^{-1}(1/2)$ <p><i>or</i> <math>\frac{1}{4}T - \omega^{-1} \cos^{-1}(1/2)</math> (M1 A1)</p> $= (\pi/6) \sqrt{(5l/7g)} \quad \text{(A1)}$	6	12
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11 (b) (i)	State both hypotheses (B0 for $r \dots$ ):	$H_0: \rho = 0, H_1: \rho > 0$	B1	4
	State or use correct tabular one-tail $r$ -value:	$r_{12, 5\%} = 0.497$	*B1	
	Valid method for reaching conclusion:	Reject $H_0$ if $0.6 >$ tabular value	M1	
	Consistent conclusion (A.E.F, $\checkmark$ on 0.497):	There is positive correlation	A1 $\checkmark$	
(ii)	Use $r^2 = ab$ to eliminate $a$ or $b$ :	$0.5 = 0.6^2/a - a$ or $b - 0.6^2/b$	M1	3
	Solve and select correct values:	$a^2 + 0.5a - 0.36 = 0$ $or\ b^2 - 0.5b - 0.36 = 0$ $(a + 0.9)(a - 0.4) = 0$ $or\ (b + 0.4)(b - 0.9) = 0$		
	(A0 if $a = -0.9, b = -0.4$ not rejected)	$a = 0.4$ and $b = 0.9$	M1 A1	
(iii)	Use $\bar{x}$ [= 5.5] in 1 <sup>st</sup> regression eqn to find $\bar{y}$ :	$\bar{y} = (66/12)b + 4.5$ [= 9.45]	M1	3
	Use $\bar{x}$ and $\bar{y}$ in 2 <sup>nd</sup> regression eqn to find $c$ :	$c = \bar{x} - a\bar{y} = 1.72$	B1	
	Sketch both regression lines on one diagram	$y = 0.9x + 4.5$ and $x = 0.4y + 1.72 \Rightarrow y = 2.5x - 4.3$	B1	
(iv)	State coefficient of $x$ in eqn of $z$ on $x$ ( $\checkmark$ on $b$ ):	$5 \times b = 4.5$	B1 $\checkmark$	2
	State value of $r$ with valid justification, e.g.:	$r = \sqrt{\{(4.5 \times (0.4/5))\}} = 0.6$ $or\ r$ is unchanged by scaling so 0.6	B1	
				12